

Gravity-like potential traps light and stretches supercontinuum in
photonic crystal fibers

The use of photonic crystal fibers pumped by femtosecond pulses has enabled the generation of broad optical supercontinua with nano-joule input energies [1, 2]. This striking discovery has applications ranging from spectroscopy and metrology [3] to telecommunication [4] and medicine [5]. Amongst the physical principles underlying supercontinuum generation are soliton fission [6], a variety of four-wave mixing processes [7, 8, 9, 10], Raman induced soliton self-frequency shift [11, 12], and dispersive wave generation mediated by solitons [6, 12, 13]. Although all of the above effects contribute to supercontinuum generation none of them can explain the generation of blue and violet light from infrared femtosecond pump pulses, which has been seen already in the first observations of the supercontinuum in photonic crystal fibers [1]. In this work we argue that the most profound role in the shaping of the short-wavelength edge of the continuum is played by the effect of radiation trapping in a gravity-like potential created by accelerating solitons. The underlying physics of this effect has a straightforward analogy with the inertial forces acting on an observer moving with a constant acceleration.

A common method of producing broad optical spectra from a spectrally narrow femtosecond pump relies upon the fact that fibers with a silica core (having few micron diameters) and surrounded by a photonic crystal cladding with various geometries, can be designed to have a zero group velocity dispersion (GVD or simply dispersion) wavelength λ_0 in a proximity of 800nm [1, 2, 6, 8], which matches the wavelength of a mode-locked Ti-Sapphire laser. Close to λ_0 the dispersion is small and so the input pulse can sustain a high peak power over a considerable length, which together with the small size of the fiber core, enhances variety of nonlinear effects, thus initiating a dramatic spectral broadening known as supercontinuum generation [1, 2, 6, 7, 8, 9, 10, 13]. The measurements of supercontinuum spectra generated with a femtosecond pump have been successfully reproduced in numerical modeling using the generalized nonlinear Schrödinger equation (see Eq. (2) in [2]). A typical example of the spectral evolution calculated using this model and leading to the supercontinuum generation by a 200fs pulse is shown in Figs. 1(a,b). The dispersion of the modeled fiber is shown in Fig. 1(c). Having λ_0 close to 800nm shifts the range of the anomalous GVD towards much shorter wavelengths than in telecom fibers and extends the range in which optical solitons can exist.

The first stage ($0 < z < 0.1\text{m}$) of the spectral broadening in Fig. 1(a) is due to the formation of symmetric spectral sidebands through the well known effect of self-phase modulation, which is mediated by instantaneous Kerr nonlinearity [14, 15]. The slope of the group velocity dispersion and its sign change do not play a significant role during this stage. At the second stage of the spectral evolution ($0.1\text{m} < z < 0.2\text{m}$) the asymmetry between the short- and long-wavelength edges of the spectrum becomes pronounced. The long-wavelength edge is shaped by the process of soliton formation (soliton fission) [6]. With further propagation the solitons are further shifted towards longer wavelengths by intrapulse Raman scattering [11, 12, 15, 16]. If dispersion is anomalous then the longer wavelengths correspond to smaller group velocities, and so the solitons continuously slow down with propagation. The spectrum at the short-wavelength edge is created by resonant radiation from the solitons [6, 13] and by the four-wave mixing of solitons with dispersive waves [7, 8]. The dispersive waves generated by both of these mechanisms initially have group velocities smaller than the ones of the solitons and lag behind the latter [8]. However, the solitons are continuously decelerated and therefore the dispersive waves catch up with and start to interact with the solitons. This interaction leads to emission of new frequencies at even shorter wavelengths [8].

Importantly, the wave packets emitted through the above process experience normal group velocity dispersion, see Figs. 1(a,c). Normal dispersion can not be compensated for by the focusing nonlinearity of the silica and hence these packets, however strong or weak, are expected to spread out in time. Therefore the efficiency of their interaction with the soliton (which is proportional to the amplitude of the wave packet [7]) should be noticeably reduced after distances comparable to the dispersion length ($\sim 10\text{cm}$) and the associated spectral broadening should cease to continue. However, experimental and numerical observations demonstrate continuous frequency shifting of the short-wavelength edge of the continuum without significant energy loss, see Fig. 1(a). Also, simultaneous spectral and time-domain analysis of the experimental and numerical data by means of the cross-correlated frequency resolved optical gating (XFROG) gives a full impression of the formation of soliton like structures not only at the long-wavelength edge of the spectrum, where dispersion is anomalous, but at its short-wavelength edge as well, where dispersion is normal and usual bright solitons can not exist, see Fig. 1(b) and Refs. [8, 17, 18]. One of the properties of the short-wavelength pulses is that their wavelength gets continuously

shorter at roughly the same rate at which the soliton wavelength gets longer. One can also see from the Fig. 1(b), that the solitons and the pulses at the short-wavelength edge are paired together, so that every soliton has an associated non-dispersive pulse on the opposite side of the spectrum.

Even without supercontinuum generation one can simply take two appropriately delayed and group velocity matched pulses with spectra across the zero GVD wavelength and observe their bound propagation, in which the dispersive spreading of the pulse in the normal GVD range been canceled and its frequency is being constantly blue shifted, see Figs. 2(a-c). This effect has been reported in the series of papers by Nishizawa and Goto [19]. Their latter work and some of the papers reporting the same effect in the context of supercontinuum generation [17, 18] have put forward the arguments that the blue shift of the pulse in the normal GVD range is caused by cross-phase modulation (XPM) mediated by Kerr nonlinearity [15]. However, the origin of the suppression of dispersive spreading, which is crucial for the continuous blue shift, has remained unclear. In what follows we demonstrate that the above supercontinuum expansion and the Nishizawa-Goto experiments are explained by the existence of a new kind of two-frequency soliton-like states. Compensation of the XPM of the short-wavelength component is one of the necessary conditions for the existence of these soliton states. This rules out XPM as a possible reason for the continuous spectral shift towards shorter wavelengths. It is instructive to remove the Raman effect from the numerical model, in which case immediate dispersive spreading of the short-wavelength component and cancelation of the frequency shifts occur, see Figs. 2(d-e). The only interaction between the two pulses remaining under these circumstances is due to XPM [15]. It is thus clear that the latter, at least in its own right, does not provide the conditions necessary for trapping and frequency conversion of the short-wavelength radiation.

The two pulse experiments and modelling, see [19] and Fig. 2, suggest that to describe continuous broadening of the supercontinuum it is sufficient to focus on the coupling between its spectral edges and neglect the middle part of the spectrum. By analyzing the numerical data one can find that the short-wavelength pulse is typically much weaker than the associated long-wavelength soliton. Therefore, only the nonlinear refractive index change induced by the soliton is significant and ought to be accounted for. The equation for the slowly varying amplitude A of the short-wavelength pulse, which is readily derived from the

generalized nonlinear Schrödinger equation, takes the form

$$-i\partial_z A - (i/v_s)\partial_t A = [d\partial_t^2 + U]A, \quad (1)$$

where $U = 2\gamma P - \tau_R\gamma\partial_t P$. The electric field of the pulse is $Ae^{ik_0z - i\omega_0t} + c.c.$, where ω_0 is the central frequency of the pulse and k_0 is the corresponding propagation constant. The notations here and below are as follows: z is the coordinate along the fiber, t is time, $v_{s,l}$ are the group velocities of the short- and long-wavelength pulses, respectively, and $d = \lambda^2 D/(4\pi c)$ is the dispersion coefficient (see Fig. 1(c) for the $D(\lambda)$ plot), P is the soliton power, $\gamma \approx 0.021/\text{W/m}$ is the nonlinear parameter [8], $\tau_R \approx 1.46\text{fs}$ is the Raman time [15]. The time dependence of the soliton power in the laboratory frame of reference is $P = 4P_0/(e^{\tau/w} + e^{-\tau/w})^2$, where w is the soliton width, $\tau = t - z/v_l + gz^2/2$ and $g = 8\tau_R\gamma^2 P_0^2/15$ is the soliton acceleration [16].

Since the Kerr nonlinearity in optical fibers is focusing the soliton created potential U locally increases refractive index. According to the Fermat principle a maximum of the refractive index serves as an attracting potential for waves experiencing diffraction. The diffraction of beams is analogous to the dispersion of pulses with a spectrum in the range of anomalous GVD. If, however, GVD is normal, then the refractive index maxima serve as repelling potential barriers and the index minima become attracting potentials for dispersing pulses. Therefore to explain the trapping of radiation at the short-wavelength edge of the continuum, we should look for a minimum in the refractive index. An alternative, but equivalent, view of the problem is to note the similarity between the right-hand side of Eq. (1) and the quantum mechanical Schrödinger equation, in which the dispersion operator ∂_t^2 corresponds to kinetic energy and the function U to potential energy. Dispersion at the short-wavelength edge of the continuum is normal, i.e., $d < 0$. Therefore Eq. (1) coincides with the Schrödinger equation for quantum particles having positive mass. Thus, U is a repelling potential for any given z (see the dashed line in Fig. 3(a)), in which case it can not trap waves and so can not lead to the formation of localized non-dispersive light pulses as observed in experiments and modeling. What this potential can do, however, is to serve as a barrier for the incident waves. This is why the dispersive spreading of the short-wavelength pulse in Figs. 2(d,e) only occurs with longer time delays. In other words, no dispersive waves can pass through the soliton.

Let us switch now from the laboratory frame to a noninertial frame of reference accel-

erating together with the soliton, i.e. together with the potential U . In order to achieve this we replace t in Eq. (1) with the variable τ (as defined above) and take account of the fact (obtained from the XFROG data) that the group velocities of the pulses at the long- and short-wavelength edges are matched, $v_s = v_l$. The possibility for this matching within the spectral range of interest is illustrated in the Fig. 1(c). Group velocities are uniquely related to frequencies through the dispersion law, and so we require that the phase of the field in the new reference frame be shifted. This is achieved by making the substitution $A = \psi(z, \tau) \exp\{i\tau gz/(4d) - ig^2 z^3/(24d)\}$ [16]. The amplitude ψ in the non-inertial frame obeys

$$-i\partial_z\psi = [d\partial_\tau^2 + U + \tilde{g}\tau]\psi, \quad \tilde{g} = -g/(4d) > 0 \quad (2)$$

Therefore the potential acting on the short-wavelength pulse in the accelerating frame has a minimum at $\tau = \tau_0$, see the full line in Fig. 3(a). The walls of the new attracting potential, $(U + \tilde{g}\tau)$, are formed, on one side, by the refractive index change created by the soliton through the Kerr and Raman nonlinearities and, on the other side, by the linear growth of the refractive index existing in the accelerating frame of reference. The linear part of the potential is equivalent in its essence to the effective gravity emulated in the accelerating reference frames and exerts a force on photons stopping them from dispersing in the direction of increasing τ . To describe the situation qualitatively it is instructive to appeal to the example of an elevator loaded with balls, see Fig. 4. If the elevator moves with a constant velocity in outer space, then the balls are weightless, they randomly kick the walls and diffuse upwards, while the elevator floor prohibits the diffusion downwards. This corresponds to the photons freely dispersing behind the soliton when the Raman effect is switched off, see Figs. 2(d,e). However, when the elevator is pulled with an acceleration, the balls pile up on the floor, just as if someone switched on gravity. This corresponds to the process of the radiation trapping with the soliton acting as the elevator floor. An analogy between the full potential $U + \tilde{g}\tau$ and the potential felt by cold atoms placed close to an atomic mirror and subjected to gravity should be mentioned also [20].

We solved the eigenvalue problem for the potential $U + \tilde{g}\tau$ numerically using a finite difference approach to determine all of its eigenmodes. We have found that the strongest soliton on the long-wavelength edge of the supercontinuum spectrum in Fig. 1(a) traps around 20 modes on the short-wavelength edge, see Figs. 3(b,c) for three examples. The potential barrier on the soliton side is high, but still finite, so that some light penetrates

through the soliton barrier creating oscillating tails. The latter are especially noticeable in the higher order modes, see Fig. 3(c). The radiation captured by the soliton can be represented as a superposition of these modes. Adiabatic transformation of the soliton power and width with propagation, caused by the increasing dispersion, induces weak adiabatic evolution of the mode parameters, but apart from this the modes are stationary solutions and hence their dispersion and spectral broadening due to XPM are suppressed. According to our approach, the phase factor $e^{-i\tilde{g}z\tau}$ used in the transformation to the accelerating frame and the evolving phases of the complex mode amplitudes should determine the spectral dynamics at the short wavelength of the supercontinuum. In order to verify this, we took the data from Fig. 1 for $z = 1.3m$ and represented the field at the short-wavelength of the continuum as a superposition of the first 10 modes (the rest can be neglected). The evolution of the trapped radiation over the remaining length of the fiber is then simply calculated by applying the appropriate phase shifts to the individual modes. The spectrum of the resulting field is again compared with the data from Fig. 1. Fig. 5 demonstrates a good agreement between the two approaches in predicting both the position of the short-wavelength edge of the continuum and its spectral shape. Small discrepancies in the shape of the spectral peaks get more noticeable for larger z and are due to the adiabatic changes of the soliton parameters, which are not accounted for in Eq. (2).

Summarizing: we have explained the physical mechanisms behind existence of the non-dispersive and continuously blue shifting localized states of light at the short-wavelength edge of an expanding supercontinuum spectrum in optical fibers. This effect has been observed in many recent experiments with photonic crystal fibers pumped by femtosecond pulses in the proximity of the zero dispersion point. We have demonstrated that the light at the short-wavelength edge of the continuum is trapped, on one side, by nonlinear refractive index variations induced by Raman solitons existing on the long-wavelength edge and, on the other side, by a linear gravity-like force originating from the fact that the soliton moves with acceleration. Our findings are an important addition to the list of physical effects known to be capable of suppressing dispersive spreading of short pulses and of transforming their frequency [15], which open new research avenues in the areas of fiber based frequency

conversion and optical solitons.

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Competing financial interests

The authors declare that they have no competing financial interests.

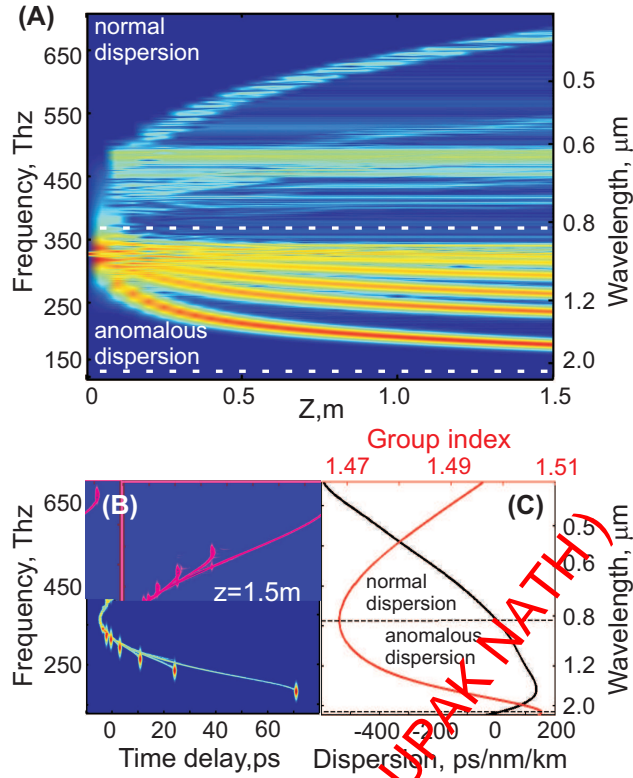


FIG. 1: **Figure 1. Numerical simulation of supercontinuum generation in a photonic crystal fiber pumped with 200fs pulses at 850nm and having 6kW peak power.** (A) Spectral evolution along the fiber length. (B) Time-frequency resolved signal for $z = 1.5\text{m}$. The localised pulses in the anomalous and normal dispersion ranges correspond to the solitons and to the trapped radiation, respectively. (C) Group index, i.e. speed of light divided by the group velocity, (red line) and dispersion D (black line) typical for photonic crystal fibers used in supercontinuum experiments [8].

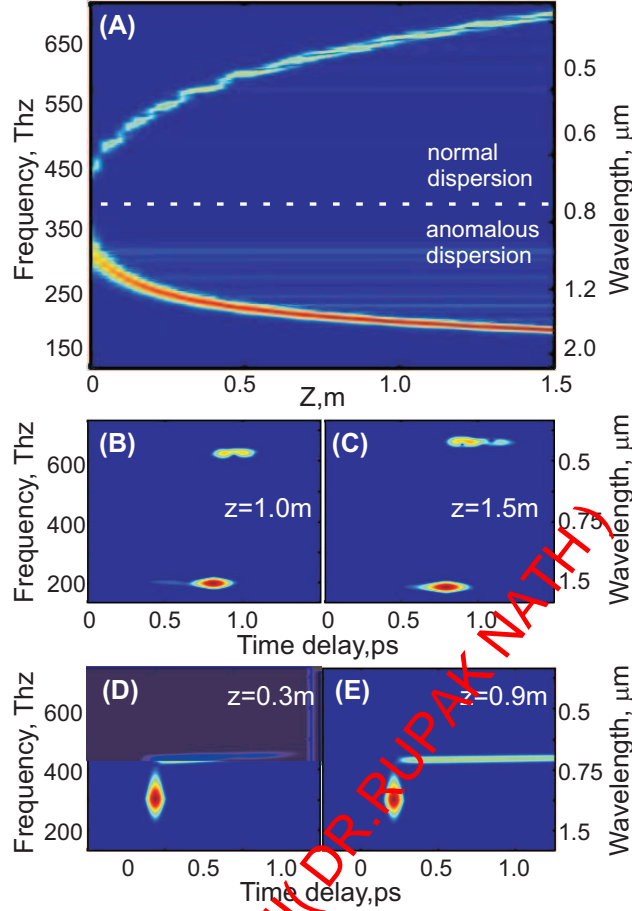


FIG. 2: **Figure 2. Radiation trapping by a soliton.** (A) Spectral evolution along the fiber length showing that the frequency of the short-wavelength radiation is continuously drifting further to the blue, simultaneously with the continuous red shift of the soliton. (B,C) Time frequency resolved signal for $z = 1\text{m}$ and 1.5m showing that the blue shifting pulse does not disperse with propagation and is delayed with respect to the soliton. (D,E) The same as (B,C), but with the Raman effect switched off. One can see dispersive spreading of the short-wavelength pulse. Parameters of the input pulses: the soliton width and peak power are 10fs and 38kW (as found from Figs. 1(B)), the short-wavelength pulse width and peak power are 30fs and 2kW. Time delay between the pulses is 50fs.

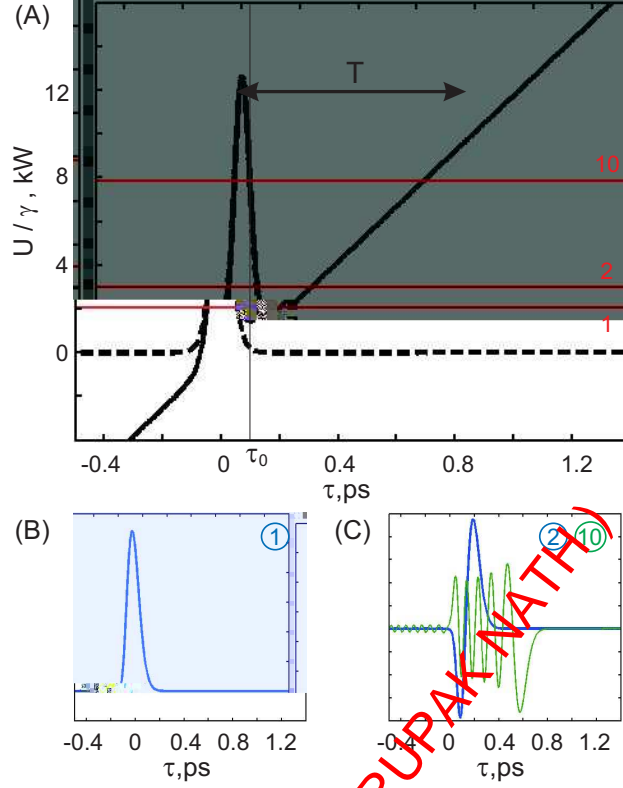


FIG. 3: **Figure 3. Effective potential and quasi-trapped states.** (A) Dashed line shows the potential U . Full line shows the potential $U + \gamma v$, which takes account of the gravity-like force. Red horizontal lines correspond to the effective 'energy levels' of the 1st, 2nd and 10th modes shown in (B) and (C).

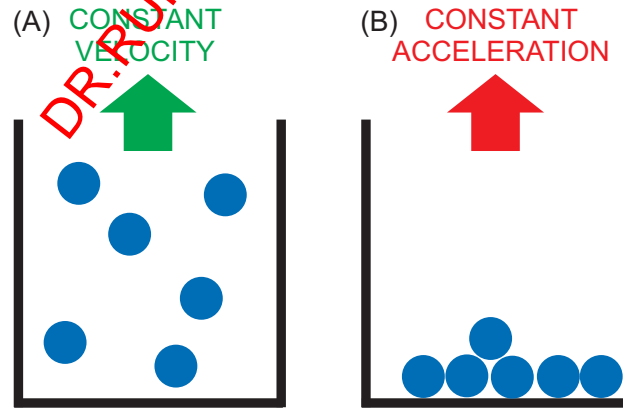


FIG. 4: **Figure 4.** Dispersion of the blue radiation in the laboratory frame, as in Figs. 2(D,E), is analogous to weightless balls flying around an elevator moving with a constant velocity, see (A). Trapped states of the blue radiation as in Figs. 2(B,C) and 3(B-E) are analogous to the balls piled on the floor of the elevator moving with a constant acceleration.

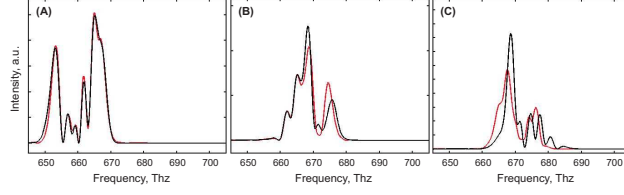


FIG. 5: **Figure 5.** The field at the short-wavelength edge of the supercontinuum can be represented as a superposition of the modes of the potential induced by the accelerating soliton, see Fig. 3. Spectral peaks at the blue edge of the supercontinuum as in Fig. 1 at different propagation distances: (A) $z = 1.3\text{m}$; (B) $z = 1.4\text{m}$; (C) $z = 1.5\text{m}$. Black and red lines correspond to the data from Fig. 1 and to the results obtained using expansion over 10 modes, respectively.

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